OptiMap Redistricting Tool

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Abstract

Drawing and evaluating political districts involves balancing population deviation while minimizing objective criteria such as the number of cut edges, perimeter length, and compactness as measured by the Polsby-Popper score. To support mapmakers, this application leverages computer optimization models to generate district plans based on user-defined specifications. The application offers two main functionalities: a guided mode and a full plan generation mode. In the guided mode, users can actively contribute to the creation of individual districts, while the full plan mode generates an entire districting plan automatically. Both modes provide 10 potential options at each step of the process. Once a plan is completed or partially generated, users can export it as a PNG image or a CSV file. The CSV file can be uploaded as a block assignment file to *DavesRedistricting.org* for further analysis or sharing.

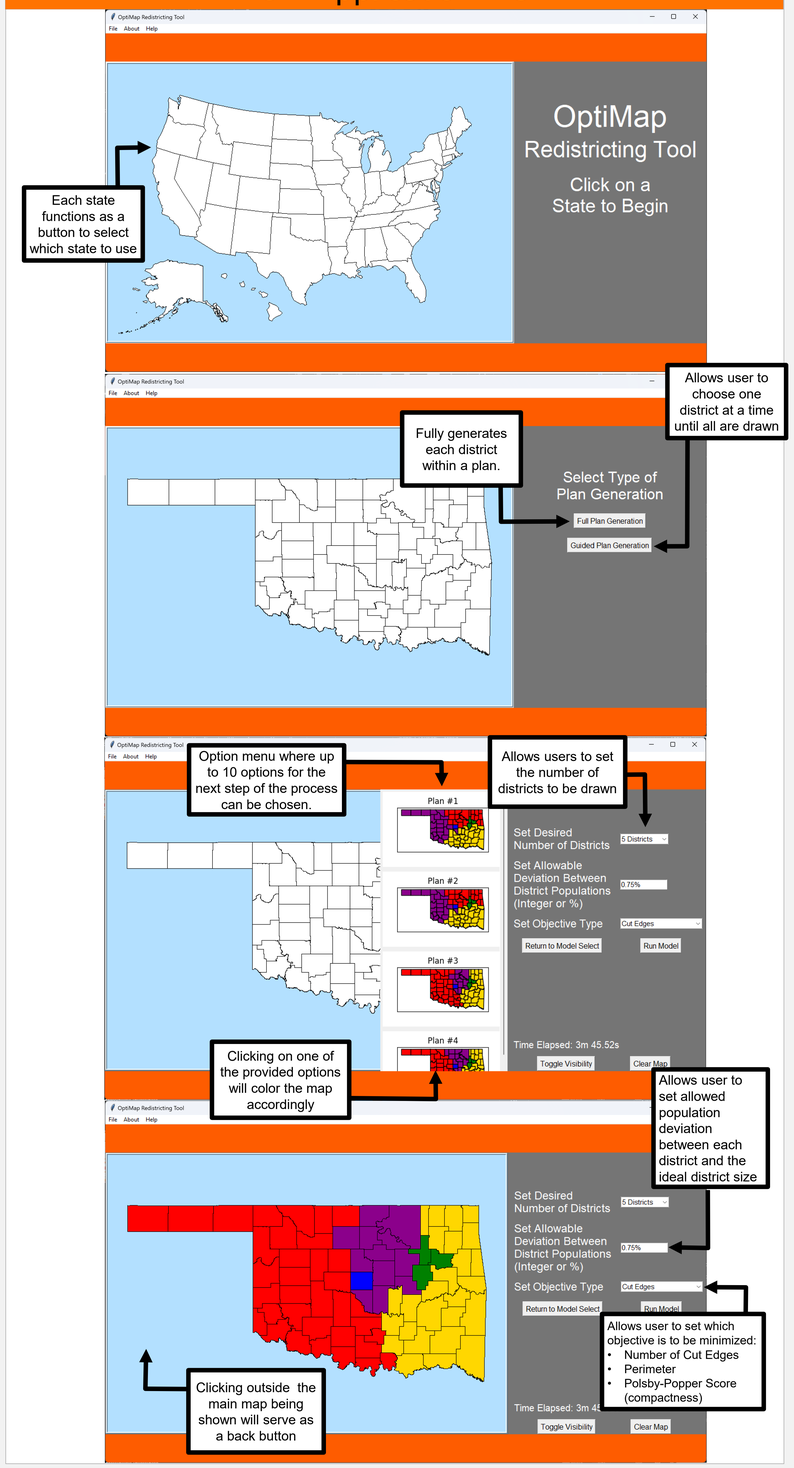
Mixed Integer Program (MIP) (Buchanan et al. 2024)

For use in the MIP, the following variable definitions will be used:

$\boldsymbol{G} = (V, E)$ is the county level graph where,
Vertices, $\boldsymbol{V}$ are representative of a state's geographic units
Edges, $\boldsymbol{E}$, pair adjacent geographic units
$\boldsymbol{r} \in V$ is a designated root county
$\boldsymbol{k}$ be the number of districts to partition the state into
$\boldsymbol{k'}$ is a designated cluster size
$\boldsymbol{p_i}$ is the population of geographic unit i, where $i \in V$
$\boldsymbol{S}$ is a subset of Vertices where $S \subseteq V$
$\boldsymbol{L}$ is the lower bound of the district population, where $L = \left\lfloor \frac{p(V)}{k} \right\rfloor$
$\boldsymbol{U}$ is the upper bound of the district population, where $U = \left\lceil \frac{p(v)}{k} \right\rceil$
$\boldsymbol{x_{ij}} = 1$ if vertex i is assigned to district j and 0 otherwise,
where $i \in V$
$j \in \{1,2\}$ which represent clusters S and $V \setminus S$
$\boldsymbol{y_e}$ is a binary variable representing each edge, $e = \{u, v\} \in E$, indicating whether it is cut
$\boldsymbol{f_{ij}}$ is a flow variable for each *directed* edge (i,j)
$\boldsymbol{N(v)}$ is neighborhood of vertex, v

$\min \sum_{e \in E} y_e$		(1a)	Minimizes the number of cut edges
$s.t. x_{i1} + x_{i2} = 1$		(1b)	Each vertex is either assigned to S or it's Complement
$Lk' \leq \sum_{i \in V} p_i x_{i1} \leq Uk'$		(1c)	Populations remain balanced
$L(k-k') \leq \sum_{i \in V} p_i x_{i2} \leq U(k-k')$		(1d)	
$x_{i1} - x_{j1} \leq y_e \text{ and } x_{j1} - x_{i1} \leq y_e$	$\forall e = \{i,j\} \in E$	(1e)	If an edge is cut, then its endpoints are either both assigned to S or neither are
$x_{r1} = 1$		(1f)	The root must be included in S
$x, y \text{ binary}$		(1g)	
$\sum_{j \in N(i)} (f_{ji} - f_{ij}) = x_{i1}$	$\forall i \in V \setminus \{r\}$	(2a)	Each vertex selected in S \ {r} consumes one unit of flow
$\sum_{j \in N(i)} f_{ji} \leq M x_{i1}$	$\forall i \in V \setminus \{r\}$	(2b)	Flow can only enter selected Vertices
$f_{jr} = 0$	$\forall j \in N(r)$	(2c)	Disallow flow from entering the root
$f_{ij}, f_{ji} \geq 0$	$\forall \{i,j\} \in E$	(2d)	
$x_{a2} + x_{b2} \leq 1 + \sum_{v \in R} x_{v2}$	$\forall a, b \in V$	(3)	Case separator Inequality for when nonadjacent vertices a,b, are disconnected when removing $R \subseteq V \setminus \{a,b\}$

Main Application View



Software Packages and Application Structure

The application was constructed using **Python** with the following packages:

Tkinter for the main graphical interface

Shapely for converting geographic data into polygons compatible with **Tkinter**

Pandas and **GeoPandas** for data management

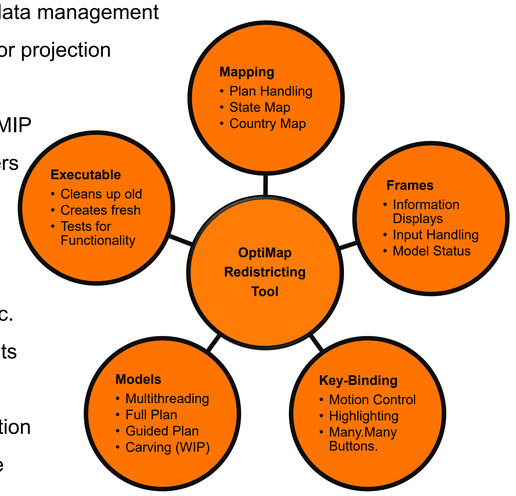
Pyproj to implement a Mercator projection on the Country level view

Gurobi Optimizer solves the MIP with user defined parameters

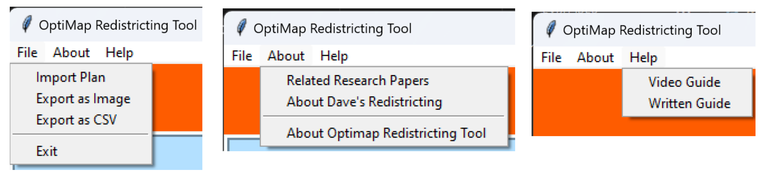
NetworkX structures census data into a graph that captures properties such as adjacency, perimeter, etc.

Matplotlib for visualizing results of the MIP

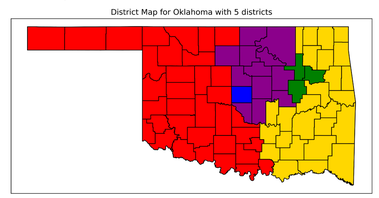
Pyinstaller to compile application as a standalone executable



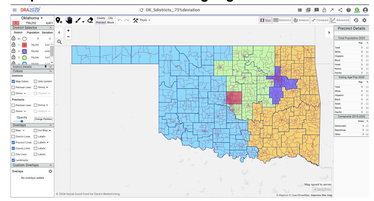
Menu and Export Options



Plan Exported as PNG



Block/Precinct Assignment File Exported to Davesredistricting.org



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